Truth and Modality - can they be reconciled?↔

by Eileen Walker

1) The central question

What makes modal statements – statements about what *might* be or what *might have been* the case – true or false? Normally a statement is deemed to be true if it describes a state of affairs which actually obtains. 'Crows are black' is true if there are black crows. But consider: 'Cameron might have lost the election'. This describes a counterfactual state of affairs – one which does not obtain. If it is true, what makes it true, and why?

2) Truth and truthmaking

First, my view of truth carries on where Peter Gibson's left off. Like him, I regard myself as a 'Rottweiler realist' (to use the expression coined by Tim Williamson). I regard coherence and pragmatist theories of truth as insufficient and as leaving the door open to antirealism.↔ The only option left seems to be some form of correspondence theory. As Peter pointed out, traditional correspondence theories, while claiming to be realist, see correspondence as a relation of equality holding between statements (propositions) on the one hand, and facts or states of affairs on the other. Correspondence is thus a symmetric relation holding between two kinds of abstract object, so is not as realist as it claims to be. Truthmaker theory attempts to address this problem. As I see it, truthmaking is an *asymmetric* relation between a statement or proposition and the way some things in the world are. The truthmaker of the statement 'There are 58 people in this room', if it's true, are the 58 people actually being in the room at the time of its utterance. A proposition or statement is true if it correctly *represents* a way things actually are. We can say that *sentences, statements, assertions* and *thoughts* all express *propositions* which represent a way the world is, in which case they are true. If the world fails to comply, they are false.

3) Truthmaking and modality

A modal proposition states what is possible, necessary or impossible. It can also assert what might be or what might have been the case. It does not state what actually *is*. And this is why modality creates a problem for theories of truth and truthmaking.

Modal statements - some examples (T = true; F = false):

1) Pigs might fly. (**F**)
2) Although David Cameron won the 2015 election (**T**), he might have lost it (**T**).
3) Although I am a human being (**T**), I might have been a poached egg or a football match (**F**).
4) The number 8 is necessarily even. **(T)**
5) The number of planets is necessarily even. **(F)**
6) There are 11 players in a football team. (**T**) It's possible that there might have been 13 (had the rules been set up differently) (**T**), but it's impossible that there could have been zero players (**T**).
7) No individual concrete object can be in two distinct spatiotemporal locations (No one concrete thing can be in two places at the same time). (**T**)
8) Necessarily, gold is the element with atomic number 79. (**T**)

We seem to have no difficulty in judging intuitively *whether* modal statements are true or false. The majority of us will agree on the *attributions* of truth or falsehood to the modal statements above. However, there are two key problems regarding the truth of modal statements. The first is a technical question: how to *account for* their truth within a logical system, so that we can assign truth values and truthmakers to them systematically. The second and deeper question is to state *why* modal statements have the truthmakers that they do have. The solution to the technical problem, 'Possible Worlds', does not address the second problem, as we shall see.

« This talk was given as a Power-Point presentation with the intention of encouraging discussion and has been adapted slightly from the original.

« To see why, see the Stanford entry, 'Truth', by Michael Glanzberg.

4) Classical logic, modal logic and truth

Classical logic is a system of representational symbols and logical operators which work together according to clearly-defined rules. The system is designed to assign truth values to statements of how things *are*. It handles statements such as 'pigs fly', and 'pigs don't fly' very efficiently, as follows:

a) 'Pigs fly' translates as 'for all x, if x is a pig, then x flies'. In symbols:[1]

$\forall x\,(Px \rightarrow Fx)$ - **False**

b) 'Pigs don't fly' translates as: 'for all x, if x is a pig, then **it's not the case that** x flies'. In symbols: $\forall x\,(Px \rightarrow \neg Fx)$ - **True**

However, classical logic is not designed to deal with how things *might be* or *might have been*. It doesn't have the means to handle modal sentences such as 'pigs **might** fly' or 'pigs **can't** fly'. So in the mid-20C a number of modal logic systems were developed to include two further operators: 'necessarily' $\Box$, known as 'box', and 'possibly' $\Diamond$, known as 'diamond'.

c) 'Pigs might fly' translates as: 'for all x, if x is a pig, then it's **possible that** x flies'. In symbols:

$\forall x\,(Px \rightarrow \Diamond Fx)$

d) 'Pigs can't fly' translates as: 'for all x, if x is a pig, then it's **not the case that it's possible that** x flies'. In symbols: $\forall x\,(Px \rightarrow \neg\Diamond Fx)$

5) Possibility and necessity defined

Necessity, possibility and impossibility are interdefinable.

A proposition which is **necessarily true** (a necessary truth) is **not possibly false**. [$\Box p \equiv \neg\Diamond\neg p$]

A proposition which is **possibly true** (a possible truth) is **not necessarily false**. [$\Diamond p \equiv \neg\Box\neg p$]

A proposition which is **actually true** is **possibly true** [$p \rightarrow \Diamond p$] - because if p weren't possible it couldn't be actual!

The axioms above make intuitive sense if we think about them, but there are problems.

6) Modality in a mess

a) The problem of substitutivity[↔]

Take the two following sentences (they are **non-modal** versions of examples in section 3):

5*) The number 8 is even.
6*) The number of planets is even.

Both 5* and 6* illustrate an important principle in **non-modal** logic: that terms which **refer to the same object** (co-referential or co-extensional terms), when substituted, should **not** affect the truth value of the sentences in which they occur. In 5* and 6* above, 'the number 8' and 'the number of the planets' are co-referential and the sentences are both true.

But recall what happens when the necessity operator is added:

5. The number 8 is necessarily even. (**True**, because it's not possible for 8 to be an odd number.)
6. The number of the planets is necessarily even. (**False,** because the number of planets could have been otherwise, and indeed was deemed to be otherwise until recently.)

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1 The logical connectives are: conjunction: ('and') &; disjunction: ('or') v; the material conditional: ('if...then') $\rightarrow$; the biconditional: ('if and only if'; '*iff*') $\leftrightarrow$; the negation operator: ('it is not the case that') $\neg$.

« Also known as the 'problem of extensionality', depending on the context of the argument.

The addition of the 'necessity' operator has failed to preserve the truth of 6*, thereby infringing the principle of substitutivity *salva veritate.* It was this problem that led Quine to reject the notion of modality altogether.

b) Structure without meaning

Various systems of modal logic are developed incorporating 'box' and 'diamond'. All of these make different basic assumptions about the interplay of necessity and possibility, and their axioms and theorems grow increasingly complex. We can end up with long strings of iterated boxes and diamonds like the following:

$$\neg((\Box\Diamond\Box\Diamond\Box\Box\Diamond\Diamond p)\ \&\ \neg(\Box\Diamond\Diamond p)$$

Most of us would have great difficulty in working out what this *means.* If we can't work out what it means, we can't tell whether it is *true*, and if we can't tell whether it is true, we can't judge whether it will play a useful role in our truthmaking theory or not. In fact, if we apply the ‘reduction rules’ of system S5 to this string of symbols, thereby eliminating all boxes and diamonds except the final ones, we get:

$$\neg(\Diamond p)\ \&\ \neg(\Diamond p)$$

This translates as: 'It's not the case that "possibly p" and "not possibly p" are both true'. In fact, it is simply the modal form of a fundamental law of logic, that a proposition and its negation cannot both be true. It is thus very useful in any theory of truth. But this is not the only difficulty with the system.

To sum up: modal logic can give strange results, can be difficult to interpret, and does not reflect everyday linguistic usage; there are several, sometimes conflicting, modal systems. In an attempt to solve these and other technical complications, Saul Kripke develops his system of 'Possible Worlds' semantics.

7) Possible Worlds and Truthmaking

In Kripke's system, developed between 1959 and 1963, Necessity and Possibility are defined as follows:

$\neg$ P: A proposition is *necessarily* true if it is true in *all* Possible Worlds.

$\Diamond$ P: A proposition is *possibly* true if it is true in *some (at least one)* Possible World.

Metaphysically speaking, a Possible World is simply a way *our* world might be or might have been. It is not an alternative concrete world, like the worlds of David Lewis. However, questions regarding the ontological status of Possible Worlds are not under discussion here. Our concern is to address the problems encountered in the previous section, and see how the apparatus of Possible Worlds can provide *truthmakers* for the modal sentences of section 3.

So can Possible Worlds explain the problem of substitutivity? 'The number of the planets is even' happens to be true in at least one Possible World - ours. But it expresses a *contingent* fact about our solar system; there are Possible Worlds where it isn't true. Setting aside taxonomical issues about the size a chunk of orbiting rock needs to reach to qualify for the label 'planet', it is surely the case that the history of the solar system might have gone otherwise. If Mars had been shattered by an asteroid, the number of planets would have been odd. So 'the number of the planets' picks out different numbers in different Possible Worlds, some odd, some even. Contrast the number 8, which in all Possible Worlds is even, never odd. Where does this leave the Principle of Substitutivity - that co-extensive expressions can be swapped around without affecting the truth value of the statements in which they occur? If there are Possible Worlds where 'the number of planets' and 'the number 8' can pick out different objects, then these expressions cannot be co-extensive after all.[↔] The Principle of Substitutivity is not infringed.

« Kripke labelled terms which pick out the same referent in all Possible Worlds 'rigid'. Rigid terms include proper names, numerals and some natural kind terms such as 'gold' or 'water'. 'The number of the planets' is a descriptive term and picks out whichever referent happens to satisfy the description. Descriptive expressions are rarely rigid.

How about truthmakers? The truthmakers of our modal statements are not the Possible Worlds themselves, but their inhabitants, if such there be. If the truthmaker for non-modal statements is a way some things in the *actual* world are, then the truthmaker for a modal statement is a way some things are in some, all, or no possible world. If 'pigs might fly' is true, there will be at least one possible world that contains the odd flying pig as truthmaker. If there is no such world, then the proposition is false. There is a possible world inhabited by a rejected and dejected David Cameron who provides the truthmaker for the assertion: 'David Cameron might have lost the 2015 election'. Gold has atomic number 79 in *all* possible worlds where the substance exists, so a proposition asserting this is necessarily true. In *no* possible world do I exist as a poached egg, so the proposition that I might have been one is necessarily false.

Now recall the two key problems raised earlier regarding the truth of modal statements. The first was technical: how to *account for* their truth within a logical system, so that truth conditions and truth values (hence truthmakers) can be assigned to propositions systematically. The 'Possible Worlds' apparatus, in combination with the systems of modal logic developed in the mid-20C, does give us a way of *applying* truth conditions and truth values to modal propositions.↔ It does *not* address the second, more fundamental question: on what *grounds* do we judge whether a modal statement is true or false? If we try to say *why* the modal statements we looked at earlier are true or false, we realise that we must *already* have some presuppositions as to how modality works *before* arriving at a judgment using the 'Possible Worlds' apparatus.

8) Grounding the truth of modal statements: the varieties of modality

Most of us would agree that pigs cannot fly, and would happily accept that there is no possible world where pigs do fly. (Claims that we could perhaps *make* pigs fly by putting them inside robot flying machines, or interfering with their genome over generations, miss the point.) But a neo-Humean might object on the grounds that we can *imagine* a flying pig, and what is imaginable or conceivable is possible.↔ If this argument seems unconvincing, it can then be objected that although it is a *physical* impossibility, there is no *logical* contradiction involved in asserting that pigs might fly. The neo-Humean is surely right on this point. But if it is logically, albeit not physically - or perhaps metaphysically - possible that pigs fly, it leads to the thought that we need to distinguish between *types* of possibility and necessity *before* we can say whether a modal statement is true or false.

Kripke and others have in fact suggested how the various systems of modal logic might map onto the varieties of necessity and possibility that we actually *use*. The commonest ones are the logical, metaphysical, physical, mathematical, epistemic and deontic modalities, each of which has its own set of Possible Worlds and key axioms. Since we are concerned with modality and *truth*, the modalities of interest are the so-called 'alethic' modalities: those where *necessarily* p implies p. (In symbols: $\Box p \rightarrow p$). If 'necessarily p' ($\Box p$) is true, it is true in *all* possible worlds. Since all possible worlds include the actual world, we would expect p to be true in the actual world. But it isn't always, as happens for example in the case of *deontic* necessity. This is the modality to do with ethics and the law, and concerns what one *ought* to do. 'Ought' ($\Box p$) implies 'can' ($\Diamond p$), but doesn't imply that p actually *happens*. In all possible worlds, one ought to nurture one's children, but there are worlds, including the actual world, where this does not occur.

So let us return to the alethic modalities, where if 'P' is *necessarily* true, it must be *actually* true.↔ The converse, clearly, does not apply: if something is *possibly* true, it may or may not be *actually*

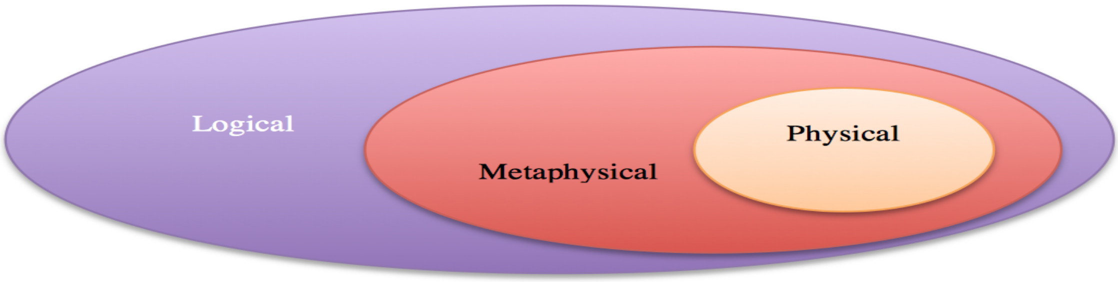
« For detail, see the Stanford entry on 'Possible Worlds'.

« The issue of whether conceivability can ground possibility is hotly contested. Most nowadays, including Stephen Yablo, say that it cannot, though David Chalmers disagrees. See their websites.

« See Peter Gibson's page on Modal Logic for a summary of how the various systems of modal logic map onto the varieties of modality, as we usually understand them. Contact him at:

true - sometimes we don't *know* enough to decide. But we should bear in mind the following constraints:

'Possibility' diagram for the alethic modalities↔



As shown in the nested diagram, it is generally accepted that *logical* possibility is the broadest species of possibility. Anything which does not contradict the laws of logic is possible. It is therefore true in some logically possible world that pigs fly, that spaceships exceed the speed of light, and that frogs turn into princes. The truthmakers for these logically possible statements are the magical beings that exist in those worlds. But these worlds are not *metaphysically* possible. Metaphysical possibilities are of course constrained by logical possibility but also by the natures or essences of all the things that exist or could exist. Physical possibilities are further constrained by the physical laws and constants of our universe.

So how does this work? A neo-Humean might claim that it is false that the element gold *necessarily* has atomic number 79. Although the proposition is true in the actual world, there might be metaphysically possible worlds where it has a different atomic number. The response to the Humean is that the natures or essences of the things that exist in our universe are subject to its physical laws and constants, and it is unlikely that they could exist in worlds where those were different - where the Big Bang had had a different outcome. An essentialist (and most chemists) would argue that a substance with a different atomic number would have a totally different nature and properties from those of gold. It would not *be* gold. So when it comes to modal statements concerning existents in the actual world, it seems plausible that physical and metaphysical possibility coincide. All contestants would agree, however, that there is no *logical* contradiction involved in the assertion that gold might have had a different atomic number.

Of course this is just the beginning of an exploration of truth and modality. I have not touched on key issues such as the epistemology of modality, the analysis of counterfactuals, possible and impossible objects, all of which are discussed in the wonderful SEP. Over to you!

References

Gibson, Peter (2015) one-page summary of modal logic. Contact him for a copy
Hale, Bob (2015) handouts from 2015 OUDCE weekend on *Necessity and Possibility*: see our website.

from the ***Stanford Encyclopedia of Philosophy (2015)***:
Glanzberg, Michael: 'Truth'; see also the articles on coherence, correspondence and pragmatist theories.
Kment, Boris: 'The Varieties of Modality'
Menzel, Christopher: 'Possible Worlds'
Vaidya, Anand: 'The Epistemology of Modality'

« Diagram borrowed from Anand Vaidya's excellent SEP article 'The Epistemology of Modality'.