Pigotts Awayday: 'What is the world *really* like, and how can we know?' – 26/7/15

(Neo-)Kantian Limits to Scientific Knowledge – Bob Clarke

NB. *This is the text I used for the presentation but with a few clarificatory additions.*

Abstract: Cogent realist philosophical arguments demonstrate that science continues to make progress in giving us knowledge of the world. But taking a Kantian 'Empirical Realist' stance, this presentation will seek to argue that, even on the basis of the most optimistic projections for continued progress, there is much that 'we' cannot know. We will draw on insights from Kant and his followers to understand why this is so, one reason being that we will only ever know the appearances, the 'Phenomena', and not the 'Things-in-Themselves': the ontology that lies behind them.

Introduction

I'm going to take a realist position that assumes that the progress of our rational and scientific enquiries gives us valid and ever-increasing knowledge of the world. But, taking a lead from Kant, I'll argue that, even on the basis of the *most optimistic* projections of the knowledge that reason and science can give us, our attainable knowledge is strictly limited[1]. Kant's philosophy is open for a realist interpretation, he stated in the *Critique of Pure Reason* (*CPR*) that his philosophical system, which he called 'Transcendental Idealism' amounted to an 'Empirical Realism' [A371][2]. But on this interpretation, Kant was the realist who maintained that we <u>*could not know*</u> what the world is really like!

Let us proceed by looking at three words in the title of this Awayday: *'World'*, *'Know'* and '*We'*.

'World' According to Kant in his *Prolegomena: 'The sensible world is nothing but a chain of appearances connected according to universal laws'.*[3] Furthermore, *'Nature considered materialiter is the totality of all objects of experience'*.[4]

'Know': Clearly there are many different forms of knowing, *knowing that*, *knowing why*, *knowing how*, etc. but, in keeping with my *'most optimistic'* approach to knowledge, I'm assuming that progress with any one variety of knowledge constitutes progress for the total sum of human knowledge. By 'knowledge' here I refer to justified beliefs about the world, or acquaintance with part of the world - where the 'justification' comes from our intersubjective (i.e. publicly shareable), informed and rationally-assessed experiences of the world, leading to an informed consensus. We'll see what Kant says about 'knowing' below.

'We': I'm going to assume that '*We*' refers to that portion of humanity who have bought into the Western-Philosophical and Scientific World-View. Essentially *'We'* are those persons who have bought-into a continuation of the <u>*Enlightenment Project*</u>. '*We*' cannot refer to all of humanity because many human groups have *not* bought-into this account of 'Knowledge'. Alternative opinions are that the only *'True'* knowledge is of the *Supernatural* or *The Mystical*!! Such groups

[1] A good introduction to Kant is Scruton, *Kant – a Very Short Introduction* (1982, 2001). A more recent introduction to his *Critique of Pure Reason* is Daniel N Robinson, *How is Nature Possible?*

[2] I'm following the usual convention in giving references to Kant's *Critique of Pure Reason* (*CPR*) by giving the pagination of the 1st German edition [A] and the 2nd German edition [B] – all modern editions have these references marked in their margins. My quotes from the *CPR* are from the Norman Kemp Smith translation.

[3] *Prolegomena* 354, according to the conventional referencing method. Page 94 in the Hackett edition.

[4] *Prolegomena* 295, Hackett: pg. 39

include religious and mystical cults who deny that *publicly available evidence from the* ***natural*** *world* has much to do with *real* knowledge. Before we reject such thinkers, we should reflect that they include in their number 'one of us' – a philosopher! Namely, Plato!!

But, to clarify, I *am* buying into the continuation of the Enlightenment Project, just as Kant did, because I believe that science gives us valid knowledge of the world. I should also make it clear that I'm *not* actually restricting my arguments to science, both good common sense and good philosophy can add to the sum total of human knowledge, but it's convenient here to use scientific examples.

Kant's *'Copernican Revolution'*

If it's true that all Western Philosophy is a footnote to Plato, it's equally true that all modern philosophy is a footnote to Kant - so many competing philosophy systems have followed in his wake that we can say that almost every extant Western philosophical movement is Kantian to some extent. So, I'm just going to pick out *the* major feature from his thought, the one he himself thought was his most important contribution to philosophy and which he referred to as his *'Copernican Revolution'* [Bxvi – xvii].

The basic idea is that we don't have *direct* knowledge of what *really* exists in the world – no direct knowledge of what he called the *'Things-in-themselves'* – the *'Dinge-an-sich'*.[5] Rather, our knowledge of the world is *mediated* by our own conceptual apparatus which he called our 'Understanding' - the innate way in which we relate to the world [B75 A51]. We can only *'know'* objects in the world as they appear to us – as *phenomena*. *Phenomenal objects* are an admixture of the Understanding that we project onto the world, and of whatever constraints the *Dinge-an-sich* place on our perceptions.[6]

Kant expressed this idea in many different *very* formal ways, but I don't want to go into Kantian terminology here – otherwise we'd be here all day! It's best to use *metaphors* to get this idea across. So we can say that *'we look at the world through human-coloured glasses'* or that our knowledge of the world is *'filtered'* and *'coloured'* by the innate *'hard-wired'* structure of our *concepts* and our *Forms of Understanding*.[7] Or that our *'Take'* on the world cannot be impartial: we see that world *always* just as humans *do*. Our knowledge is merely *perspectival*.

One thing Kant did say which is useful is that *'reason has insight only into that which it produces after a plan of its own'* [Bxiii], so the *phenomena* that we generally suppose that we *take in* from the world, we, in fact, *pro-actively* help to create. The consequence is that the basic ontology of the world is not available to us! We can only *'know'* phenomena and not the *Dinge-an-sich*, which constitute the real ontology of the world.[8] Or to put this another way, *'knowledge'* is a relationship in which we can stand with respect to the *phenomena*, but not towards the *Dinge-an-sich*.

Kant on Knowledge

[5] See e.g. *CPR* [Bxxvi], but *passim* throughout the *CPR*.

[6] Note that, on a realist reading of Kant, the *Dinge-an-sich* actually *do* affect us, do interact with us and do constrain our perceptions of the world. But the mode of that interaction is obscure to us and it cannot deliver us *knowledge* of the basic ontology of the world – for reasons given below. We can reject this realist reading but then we would no longer be realists! My reasons for being a realist are non-Kantian & are similar to Peter Gibson's. I've deliberately missed out any references to Kantian *noumena* in this presentation, because they will just open out too many issues: e.g. an alternative 'anti-realist' reading of Kant.

[7] Kant's '*Forms of Understanding'* are explained in his *'Transcendental Aesthetic'* [A19 B33 ff]

[8] See [Bxx] *' … knowledge has to do only with appearances, and must leave the thing in itself as indeed real per se, but as not known by us.'*

The whole of Kant's *First Critique* (the *CPR*) can be seen as Kant's answer to *'What can we know?'*. Kant often takes on the mantle of an Empiricist when he talks about knowledge. At the very start of the *CPR*, the very first sentence of the Introduction of the 2nd Edition in the Norman-Kemp-Smith translation, he states: *'There can be no doubt that all our knowledge begins with experience.'* – **FULL STOP!** [B1]. How nice! Here is Kant being succinct and clear for a change! But, if we turn to the German original, the full-stop is actually a semi-colon & the sentence continues for another six clauses!![9]

What's more, the German word that Kant uses here that Kemp-Smith (in 1929) translated as 'knowledge' is actually '*Erkenntnis'*, which later English editions translate as 'Cognition'. The word '*Erkenntnis'* is based on the German verb '*kennen'*, best translated as 'to be acquainted with' and we should note that most other European languages, apart from Southern English, distinguish between knowing *intellectual facts* (*'wissen'* in German, *'savoir'* in French) and acquaintance with *states of affairs* or *persons* (*'kennen'* in German, *'connaître'* in French). Indeed Northern English makes that same distinction by using the word 'ken' for the latter – *'D'ye ken John Peel'* - which has the same etymological root as the German word. We are reminded yet again that there are many different forms of knowledge, and Kant is actually talking about 'acquaintance' or 'cognition'. This is the knowledge-faculty we need to relate to the phenomenal world.

So for Kant the *Dinge-an-sich* and the basic ontology of the world are literally ***'beyond our ken'.***

Updating Kant

But Kant was writing over 200 years ago and things have moved on since his time! In Kant's day human perception via the electromagnetic spectrum was limited to light frequencies, we now use the whole spectrum from sub-Hertz frequencies, via radio, microwave, infrared, light and ultraviolet up to Gamma-Ray frequencies to interrogate the world. Indeed we can throw other things at the world - like protons, as they do in the Large Hadron Collider at CERN, and we can zap the world with other projectiles like sound (phonons) too, and then we can study what comes off it.

Surely, today, we have far better knowledge of the world than Kant could ever have attained because we can access it in so many more ways!? Well, yes, indeed we do! Furthermore, by working together in, say, research institutions, and by using the principle of the division of labour (each scientist has a specialism), we can discover facts about the world far more efficiently than in Kant's day. Science has made very real progress, and our knowledge of the world has expanded enormously because of it. We need have no doubt about that. Given suitable encouragement (e.g. funding), it will continue to do so. **BUT!** As I've said, my neo-Kantian thesis is that, even if we stack all the cards in favour of our optimistic realism, and of continued progress of rational thought, our knowledge of the reality behind the *phenomena* will remain strictly limited.[10]

So I want to give you **four arguments**, based on our contemporary knowledge of the world, as to why Kant was probably right about this.

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[9] Actually Kant's next clause states: *' … for how should our faculty of knowledge be awakened into action did not objects affecting our senses partly of themselves produce representation …,'* – which can certainly be given a realist connotation – he goes on to expound his Copernican Revolution.

[10] Here are a couple of things that Kant says about this: *'It is not that by our sensibility we cannot know the nature of things in themselves in any save a confused fashion; we do not apprehend them in any fashion whatsoever'* [A44 B62]! *'… nothing whatsoever can be asserted of the thing in itself …'* [A49 B66] – which I think is going a little too far!

But, to keep a balance, I'll point to a couple of arguments for doubting Kant before we start:

> **Number:** If we catch just one electron in an ion trap, surely we *know* that there is just one *Ding-an-sich* behind the phenomenal electron that we have captured? Or, to turn the argument around, and as Schopenhauer pointed out, if we know strictly <u>*nothing*</u> about the *Dinge-an-sich*, how did Kant know that there is more than one of them? Schopenhauer argued that if we cannot know that the *Dinge-an-sich* are plural, we should only refer to the concept holistically - one big *Ding-an-sich*, which he went on to call *The Will*!
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> **Space:** In what sense are we ontologically *wrong* about our place in the Universe? It seems so apparent *phenomenally* nowadays that we live on a ball of rock circling a star in a galaxy in a vast universe, that we are inclined to believe that this statement must capture some *ontological* reality.

Well maybe – it's often difficult to accept Kant's pessimistic view - but let's canvas some thoughts as to why Kant might actually be right.

Kant, in his own day, would *decidedly not* have proffered the four examples I'm about to give, but he was always keen to keep up with the progress of science and he invited us to take his philosophy forward in the light of the ongoing progress of human knowledge, which is what I'm doing here.[11]

Four Arguments to the Effect that our Possible Knowledge must be Limited

1. Quantum Mechanical 'Interpretations'

The 'new' Quantum Mechanics (QM), which we still use, was developed by a number of philosophically astute physicists in the 1920s. They were always aware that QM was a theory about *appearances* - about *phenomena* – their so-called *Copenhagen Interpretation* of QM was philosophically explicit about this.[12] Quantum Mechanics is one of the most successful physical theories of all time for predicting *phenomena*. Indeed in its most developed form, *Quantum Field Theory*, it is arguably ***the*** *most* successful physical theory of all time, making predictions of a number of atomic constants to better than parts in a billion.

The 'Interpretations' of Quantum Mechanics are putative metaphysical explanations for this literally *'phenomenal'* ability of QM to predict *phenomena* so accurately. Some of these interpretations attempt to provide a *realist ontology* that explains the *phenomena* – essentially to give accounts of *Dinge-an-sich* that lie behind the *phenomena*. We shall look at just two such ontological interpretations: the ***Many Worlds*** and the ***Transactional Interpretation.***[13]

> **The Many Worlds Interpretation** maintains that whenever the mathematical formalism of QM gives rise to statistical probabilities that one of a number of outcomes could come about, the world actually splits into multiple branches – each of the possibilities is ontologically instantiated in at least one of these branches, and these branches *really do* exist. Schrodinger's Cat can be used to give an example of this approach.[14]

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[11] In [Bxliv] Kant bequeaths the continuation of his work to later thinkers. His Latin quote from Francis Bacon [Bii] has a similar intention.

[12] The *Wikipedia* entry on the Interpretations of Quantum Mechanics gives a pretty good idea of what is going on here.

[13] See Gribbin, *Schrödinger's Cat*, and Gribbin, *Schrödinger's Kittens*, respectively.

[14] See Gribbin, *Schrödinger's Cat* again.

The supporters of the **Transactional Interpretation** regard *Many Worlds* as profoundly and unnecessarily profligate. It claims that, ontologically, there is *only one single time-line* for the world, but that its physics is held together by means of signals travelling forwards *and backwards* in time. Future events are effectively made compatible with past events by resonances between them through space-time.

The supporters of each of these and other ontological interpretations of QM seek to undermine their rivals by claiming that opposing positions are philosophically incoherent – and indeed all ontological interpretations have profound metaphysical difficulties.[15] But on the *charitable* assumption that our two Interpretations are both viable, *you don't need me to tell you* that a world which is constantly splitting into a potentially infinite set of branches and in which signals can only travel forward in time is *totally different* ontologically from world with a non-branched single time-line in which signals can travel backwards and forwards in time!

Now, the *important point* here is that these interpretations **are *intentionally* experimentally indistinguishable**! They predict the *same phenomena*, otherwise they do not do their job properly! All this goes to show that modern science, in the form of QM, predicts the phenomena brilliantly, but it ***totally underdetermines*** the ontology behind the phenomena – just as Kant claims. At the Awayday Mike Arnautov summed this up very succinctly in the phrase:

Phenomenology always underdetermines Ontology.[16]

All of which is a rather roundabout way of saying that theory is always underdetermined by evidence from a finite number of facts – which is all we ever have in science![17]

2. What is Time?

Kant maintained the *Ideality of Time*: that Time is one of the *appearances* that we impose upon the world.[18] While he maintained that there may be ontological prototypes for phenomenal *objects* – the *Dinge-an-sich*, there is no such ontological prototype for Time – it is entirely imposed upon the world by our Understanding.[19] Given the importance of Time in our lives, this has always been a hard claim to swallow! Surely Time is real! But we still do not know what Time is! A group of scientists and philosophers have set up a *Foundational Questions Institute* (FQXI) which addresses just the sort of issues that we are tackling today.[20] Each year they run an essay competition that seeks answers to fundamental questions.[21] For example:

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[15] For example, the supporters of *Many Worlds* claim that the *Transactional Interpretation* fails to describe the QM of more than one particle! But supporters of the latter are undeterred and continue to develop it: see Ruth E Kastner, *The Transactional Interpretation of Quantum Mechanics: The Reality of Possibility*, (Cambridge: 2012).

[16] Thanks Mike.

[17] See the *Stanford Encyclopedia of Philosophy* or *Wikipedia* on the *Underdetermination of Scientific Theory*.

[18] See *The Transcendental Aesthetic* in the CPR but especially [A30 B46 ff]

[19] Note that being a realist does not disallow one from asserting that *some* parts of our experience are ideal, e.g. an inveterate 21st Century realist may well hold that 'love' is purely ideal. Kant thought that space and time were ideal, but he was definitively *not* an idealist: see [B274-B279] and [A367] for his *Refutation of Idealism*. In [A39 B56 ff] he argues that *'... the ideality of space and time leaves, however, the certainty of empirical knowledge unaffected'*.

[20] See http://fqxi.org/

[21] All of the winning essays can be downloaded from: http://fqxi.org/community/contest/

2010-11: *Is Reality Digital or Analog?*
2012: *Which of our Basic Physical Assumptions are Wrong?*

In 2008 the essay question was about the *Nature of Time*. Answers came from leading scientists and philosophers, Sean Carroll, Carlo Rovelli, George Ellis, David Hestnes, Julian Barbour, Steven Weinstein - FQXI does not pander to uniformed bloggers (like much of the rest of the Internet!).[22] No two of these great thinkers agreed on what Time was – their accounts were radically different.

But Time, and its inverse, Frequency, are the quantities that metrologists can measure most accurately in the world, so *it's ironic in the extreme* that, with each century, we generate ever more ontological interpretations of what Time actually is! Instead of homing in on one favoured ontology, a *consensus*, which is what we hope for from the progress of knowledge, we are creating an ever-growing plethora of interpretations.[23] Does this constitute a growth of knowledge? Or does it indicate that we are dealing with a concept whose ontology we cannot grasp!? That Time itself is ***beyond our ken***? It was precisely the myriad opinions of medieval Scholastics regarding supernatural issues that prompted the Eighteenth Century Enlightenment thinkers – especially Kant – to inveigh against such opinions and to conclude that we can have no knowledge of supernatural matters. Don't we have a similar situation here regarding the ontology of Time?

As for Kant's *Ideality of Time*, an equation deriving from the advanced theory of Quantum Gravity, the Wheeler–deWitt equation, arguably indicates that time is not a *fundamental* feature of the world, time cannot be defined at the most basic level of the world![24] For this reason Julian Barbour in his winning FQXI essay and in his book claims that 'time is an illusion'. Likewise **Claus Kieffer** (a leading physicist) argues in his winning FQXI essay that time only emerges from the separation of the world into different subsystems. But this separation is just the kind of thing that <u>we</u>, in our *analytical* attempts to understand the world scientifically, impose through our understanding of the world. Do we thereby create Time? Is Time therefore ideal? Was Kant right all along!?

3. Physical Models and Paradigms

One manifestation of our post-Kantian philosophies of science is their hypothetico-deductive nature. Intellectually, we impose structures of laws and postulates upon the world to give us an understanding of it. We label these structures as 'theories' or 'paradigms'. This is quite explicit in contemporary fundamental physics. We no longer take our theories to be 'true', we speak of them as 'formulations' or 'hypotheses' or 'models'.[25] They are meant to give us *perspectives* on the world that are pragmatically useful. Of course we don't do this at random, theories are empirically tested and are only adopted if they are useful. Each such theory gives us a *perspective* on the world which we understand intellectually. Modern science is not *monolithic*, it is a patchwork-quilt of such theories and paradigms which are tools for solving different types of problems.
Now, since Kant's time we have likewise come to understand that the human psyche is not *monolithic* either. We develop some of our understandings of the world via our intellectual faculties. But they appear, phenomenologically, to lie in a different part of our mind from that in which we hope to get a real *gut knowledge* of the world that we can be really satisfied with. Arguably, it was the latter that Kant was originally concerned with. We readily accept that the scientific theories that we explicitly impose on the world give us limited *perspectives* and *prescriptions* which allow us to

[22] All of the winning essays on *'The Nature of Time'* can be downloaded from: http://fqxi.org/community/essay/winners/2008.1
[23] There are number of popular books on these issues, see Barbour *The End of Time*, Smolin, *Time Reborn* and Unger and Smolin *The Singular Universe*
[24] See Barbour *The End of Time*. The *Wikipedia* entry rapidly gets very technical!
[25] Note 'hypo-theses' = 'under-theses': at best *'underlying a thesis'*, at worst *'not as good as a thesis'* – pretty dodgy!

relate to the world. We can metaphorically assimilate them to so many '*soft*-ware' algorithms which we can choose between. We should likewise recognise that the in-built organic mental 'hard-ware' or rather 'wet-ware' (well, arguably, our brain-based 'somewhat-damp-ware') algorithms that nature has bequeathed to us are equally *perspectival* and *prescriptive* in their operation and so, likewise, are model-like and limited in their capacity to *know* the *real* world. There are no *naturally*-based arguments to the effect that they are exempt from such limitations. What would they consist in? Only *super-natural* arguments would suffice & I'm discounting them today (as I always do).

4. Creature Knowledge

There is considerable work going on between philosophers and scientists these days concerned with how animals, and indeed other creatures, *cognise* the world.[26] The word 'cognition' in these studies is extended beyond *conscious* cognition to the more prosaic question 'what does the creature need *to be acquainted with* in its environment in order to subsist?', 'what does it need to be aware of?'. For an amoeba the answer is 'very little'. It needs to know about its nutrients – it naturally navigates up a sugar gradient because sugar is what it eats. It might avoid a few things like extreme cold, but beyond that it's knows very little about the world. Most of the things that we 'clever' humans know about its environment are literally ***beyond its ken***!

Moving up the scale of sophistication in life we might think about cats, dogs or horses. In fact, consider *Champion the Wonder Horse*! Champion is a very clever horse, who knows quite a lot of horsey things and, because of his long association with humans, he knows a few human things too. He knows how to count up to 12 with his hoof. He knows how to save the lives of innocent people when they are tied to railroad tracks in the path of an oncoming train by dastardly villains! *But you can bet your bottom dollar that he doesn't know that there are Cepheid Variable Stars in the Andromeda Galaxy!*

Animal knowledge of the natural world can be seen to be limited. Human Beings are animals. It would take *an act of extreme hubris* to claim that we are not similarly limited in our knowledge by our very nature - that humanity is exempt from the cognitional limitations exhibited by other species. We have every reason to see that our knowledge is yet again *perspectival* and *prescriptive*: we see the world *partially*, as human beings see it, and no more. This fact limits the range of our possible knowledge. (Indeed, we may even wish to argue that other species can know the world in ways we cannot. *What's it like to be a bat?*).[27]

Summary

My four arguments present complementary philosophical perspectives on my claim that our knowledge of the Natural world is in principle limited. But they constitute *rather radica*l updates of Kant's approach - he would probably disapproved of them. Kant used a language of *logic* regarding our mediated innate understanding of the world. He spoke of *a priori*, apodictic and analytic

[26] In one direction, this takes us into theories of 'Autopoiesis' and 'Enactivism': see e.g. Evan Thompson, '*Mind in Life'*, (Harvard U P, 2007) and Andreas Weber and Francisco J. Varela, *'Life after Kant: Natural purposes and the autopoietic foundations of biological individuality'*, *Phenomenology and the Cognitive Sciences*, **1:** 97–125, 2002. Kluwer Academic Publishers.

[27] We might wish to argue that a Genetic Engineering programme to generate 'Trans-humans' would be able to give *'Us'* access to still more knowledge of the world. Maybe so, but this is where my demarcation of who *'We'* are on page 1 is important. I'm assuming that *'We'* are *human*, so such extended knowledge will not be available to '*Us*'.

propositions about the structure of our understanding. These days we may be more inclined to use a more organic language. He thought that all rational beings would understand the world in the same way, whereas these days we're more inclined to relativize understanding to each species (and indeed to each human culture and to each paradigm). But the basic message is the same: ***we*** impose the structures of our understanding on the world. Putting it strongly, ***we*** *contaminate* our world with the nature of our understanding. We do not see the bare naked *Dinge-an-sich*. We are *not* in a position to know them.

Clearly, I've covered only knowledge of the *natural* world, where we know only the *phenomena*, not the *ontology* behind the *phenomena*. Of course, *a fortiori*, Kant denied that we have any knowledge **at all** of the *supernatural* world![28]

But, finally, we may note that the ancient Greeks exhorted us: *'Know thyself'*. I've argued that we have no guaranteed knowledge of *Things-in-themselves*. But Kant refers to human beings as *Ends-in-Themselves*, and Kant's arguments apply equally to them. For Kant, *we do not even know ourselves*, only a mediated construction of ourselves![29]

References

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R M Unger and Lee Smolin, *The Singular Universe and the Reality of Time*, (Cambridge, 2015)

[28] See his *Transcendental Dialectic* in the *CPR*: [A293 B349 ff] which deals with the *Logic of Illusion* which arises when we have no empirical evidence for our opinions.

[29] See [Bxxvii – Bxxviii], [B154-B159]. For *Ends in Themselves*, see Kant's *Groundwork for the Metaphysics of Morals*.